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Robust Optimal Control of Chaos in Permanent Magnet Synchronous Motor with Unknown Parameters

This paper focuses on the problem of chaos suppression in permanent magnet synchronous motors (PMSM) with uncertain parameters. Firstly, the chaotic characterizations are analyzed, including bifurcation diagram, chaotic attractor, Lyapunov exponents, and power spectrum. Then, by using robust optimal control approach, a simple linear feedback controller is designed to make the system states stable. And the control gains can be obtained from a linear matrix inequality (LMI), which can be resolved easily via the MATLAB LMI toolbox. Based on Lyapunov stability theory, the stability of the proposed scheme is verified. Finally, numerical simulation results are given to demonstrate the effectiveness of the presented method.

Keywords: Permanent magnet synchronous motor (PMSM), robust optimal control, linear matrix inequality (LMI), unknown parameters.

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1. Introduction

Since the late 1980s, chaos has been found in all kinds of motor drive systems, such as induction motors, DC motors, and switched reluctance motors[1]. Chaotic behavior in permanent magnet DC motor was first investigated by Hemati[2]. Li[3] identified that chaos was also existed in permanent magnet synchronous motor(PMSM). Without considering power electronic switching, a PMSM drive system can be transformed into a typical Lorenz system, which is well known to exhibit Hopf bifurcation and chaotic behavior. This behavior, which will strongly affect the performance of the PMSM drive system, is undesirable in many fields. Thus, chaos control in PMSM has become a very important topic during the last decades.

Up to now, various methods have been investigated to stabilize the PMSM chaotic system, including delayed feedback control[4], passivity control[5], inverse system control[6], nonlinear feedback control[7], Lyapunov exponents method[8, 9], differential geometry method[10], and other methods. However, most of those methods are valid only for the system whose system parameters are precisely known. In fact, some parameters of PMSM chaotic system, such as resistance, inductance and flux linkage, are probably unknown and may change over time. Thus, adaptive control has been used to control chaos in PMSM[11, 12]. Wei [13] has combined passivity theory and adaptive control to achieve the robust stable of the system with uncertain parameters. Yu [14] has designed an adaptive fuzzy controller to achieve the precision position tracking control of the chaotic PMSM system. . Recently, several other methods have also been successfully utilized for stabilizing PMSM chaotic system with parametric uncertainties, such as sliding mode control[15], finite time control[16], impulsive control[17] and fuzzy control[18]. However, none of the aforementioned methods have taken into consideration of the problem of the control effort required to stabilize PMSM chaotic system. An effective method to deal with this problem is linear quadratic (LQ) optimal control, which can not only satisfy physical constraints (for example, the actuator will eventually saturate), but also minimize some performance measures simultaneously. This method was first introduced by Lenci and Rega for

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controlling a discontinuous chaotic system[19]. Then Lenci developed this method to control a continuous Helmholtz oscillator[20]. Recently, several researchers have applied this approach to control and synchronization of chaotic and hyperchaotic system [21-26]. To the best of our knowledge, the optimal control problem of chaotic system with uncertain parameters has not been studied yet.

This paper presented a robust controller based on optimal control approach and LMI technique to improve the stable performance of PMSM chaotic system with uncertain parameters. The merits of the presented scheme are that not only the global stable is obtained but also the optimal and robust performance is achieved in the controlled PMSM chaotic system. The simulation results for PMSM chaotic system demonstrate the effectiveness of the proposed scheme. The remainder of this paper is organized as follows. Section 2 introduces the chaos model of PMSM and gives the problem formulation for chaos control of PMSM. Our main results and the realization of chaos control are described in Section 3. In Section 4, some simulation results are provided to illustrate the effectiveness of the proposed method. Finally, we summarize this paper in Section 5.

Notations: R^n and $R^{m\times n}$ denote the set of real numbers, the n-dimensional Euclidean space and the set of all real $m\times n$ matrices, respectively. $\|\cdot\|$ denotes the Euclidean norm. $\lambda_{\min}(\cdot)$ and $\lambda_{\min}(\cdot)$ represent the minimal and maximal eigenvalues of a matrix, respectively. represent the trace of a matrix. The symbol M^T and $*$ represent the transpose of a matrix M and the transpose elements in symmetric positions. $I^{n\times n}$ is an n-dimensional identity matrix.

2. Chaos in PMSM and problem formulation for chaos control

2.1. Chaos in PMSM

The transformed model of PMSM with the smooth air gap can be expressed as follows[3]:

$$\begin{cases} \dot{i}_d = -i_d + wi_q + \tilde{u}_d \\ \dot{i}_q = -i_q - wi_d + \gamma w + \tilde{u}_q \\ \dot{w} = \sigma(i_q - w) - \tilde{T}_L \end{cases} . \tag{1}$$

Where $\tilde{u}_d$, $\tilde{u}_q$, i_d and i_q are the transformed stator voltage components and current components in the d-q frame, ω and $\tilde{T}_L$ are the transformed angle speed and external load torque respectively, γ and σ are the motor parameters.

Considering the case that, after an operation of the system, the external inputs are set to zero, namely, $\tilde{u}_d = \tilde{u}_q = \tilde{T}_L = 0$, system (1) becomes an autonomous system:

$$\begin{cases} \dot{i}_d = -i_d + wi_q \\ \dot{i}_q = -i_q - wi_d + \gamma w \\ \dot{w} = \sigma(i_q - w) \end{cases} . \tag{2}$$

The modern nonlinear theory such as bifurcation and chaos has been used to study the nonlinear characteristics of PMSM drive system in [3]. It has found that , with the operating parametersγ and σ falling into a certain area, PMSM will exhibit complex dynamic behavior, such as periodic, quasi periodic and chaotic behaviors. In order to make an overall inspection of dynamic behavior of the PMSM, the bifurcation diagram of the angle speed w with increasing of the parameter γ is illustrated in Fig. 1 (a).We can see that the system shows abundant and complex dynamical behaviors with increasing parameter γ . The typical chaotic attractor is shown in Fig. 1 (b) with $\tilde{u}_d = \tilde{u}_q = \tilde{T}_L = 0$, $\gamma = 25$, and $\sigma = 5.46$.

According to chaos theory, the Lyapunov exponents and power spectrum are two effective methods to determine whether a continuous dynamic system is chaotic. In general, a three-dimensional nonlinear system has one positive Lyapunov exponents, implying that it is chaotic. Fig. 1 (c) and (d) show the Lyapunov exponents and power spectrum of PMSM chaotic system (2) with $\gamma = 25$, and $\sigma = 5.46$. When the parameters are set as obove, calculated Lyapunov exponents are: $L_{E1} = 0.479453$, $L_{E2} = -0.024905$, $L_{E3} = -7.914548$,and the Lyapunov dimension is $D_L = 2.057432$, which means the system is chaotic.

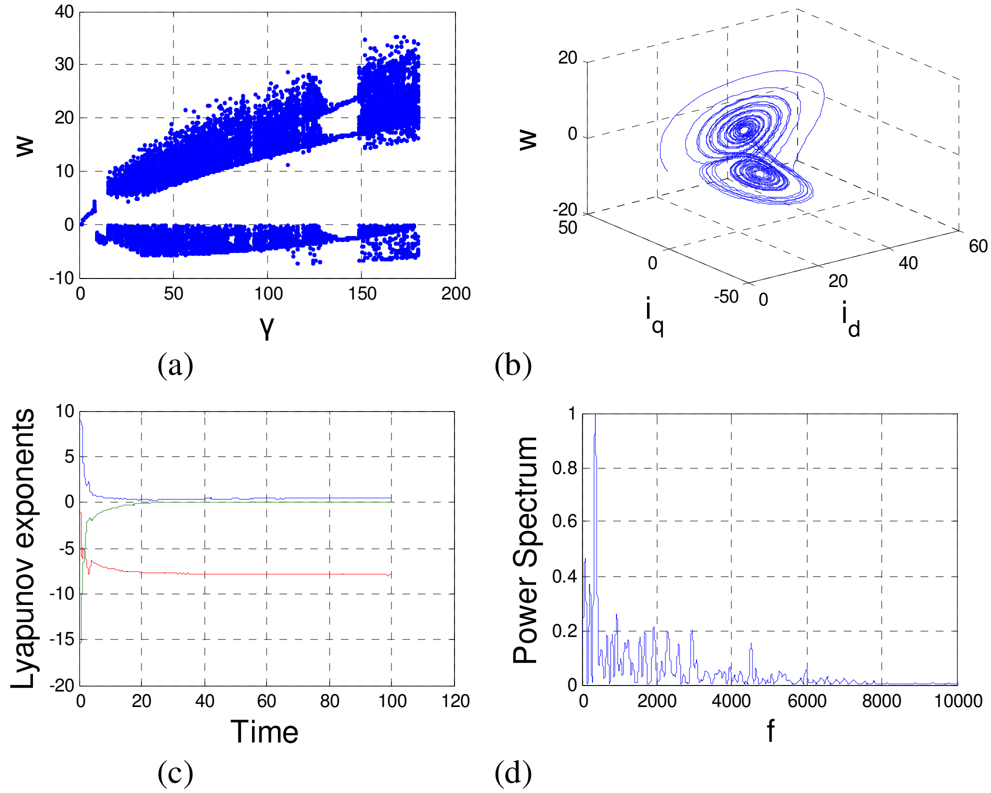


Fig. 1 Bifurcation deagram and the Characterizations of chaos in PMSM (a) Bifurcation diagram of state variable ω with the parameter γ (b) typical chaotic attractor (c) Lyapunov exponents (d) power spectrum of state variable ω

2.2 Problem formulation

Considering system uncertainties and adding the control inputs, system (2) can be described as:

$$\begin{cases} \dot{i}_d = -i_d + wi_q + u_1 \\ \dot{i}_q = -i_q - wi_d + (\gamma + \Delta_\gamma)w + u_2 \\ \dot{w} = (\sigma + \Delta_\sigma)(i_q - w) \end{cases} . \tag{3}$$

Where Δ_γ and Δ_σ represent the uncertainty of γ and σ respectively and the upper bound of Δ_γ and Δ_σ are known, u_1 and u_2 are control inputs.

Following an actual operation, this article assumes that the fluctuation range of system parameters is 30%, that is, $\|\Delta_\gamma\| \le \delta_1 \le 0.3\gamma$, $\|\Delta_\sigma\| \le \delta_2 \le 0.3\sigma$.

System (2) indicates three equilibrium points: $S_0(0,0,0)$, $S_1(\gamma-1, \sqrt{\gamma-1}, \sqrt{\gamma-1})$, and $S_2(\gamma-1, -\sqrt{\gamma-1}, -\sqrt{\gamma-1})$. Given that $\gamma = 20$, S_0 is locally stable, and S_1 and S_2 are both locally unstable[3]. Without loss of generality, we select the origin $S_0(0,0,0)$ as the desired equilibrium point. The other non-zero equilibrium points S_1 and S_2 can be translated into S_0 by a simple coordinate transformation.

In order to design the controller, system (3) can be rewritten in compact form as:

$$\dot{x} = (A_0 + \Delta A)x + g(x) + Bu \ . \tag{4}$$

Where $x=(x_1,x_2,x_3)^T=(i_d,i_q,\omega)^T$, $A_0=\begin{bmatrix}-1 & 0 & 0\\ 0 & -1 & \gamma\\ 0 & \sigma & -\sigma\end{bmatrix}$, $\Delta A=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & \Delta\gamma\\ 0 & \Delta\sigma & \Delta\sigma\end{bmatrix}$,

$B=\begin{bmatrix}1 & 0\\ 0 & 1\\ 0 & 0\end{bmatrix}$, $g(x)=\begin{bmatrix}x_2x_3\\ -x_1x_3\\ 0\end{bmatrix}$, $u=(u_1,u_2)^T$.

Associated with system (4) is a quadractic performance index

$$J(x(t),u(t))=\int_0^\infty (x(t)Qx(t)+u(r)Ru(t))dt \ . \tag{5}$$

Where Q and R are the given positive definite sys-metric matrices.

The objective of robust optimal control is to find an linear feedback controller u that drives the system(4) from any initial state to desired fixed point S_0 with minimizing the performance index (5).

3. Main results

Considering the following affine nonlinear system with the similar structure to system (4):

$$\dot{x}(t)=(A_0+\Delta A)x(t)+g(x)+Bu(t),\ x(0)=x_0 \ . \tag{6}$$

Where $x(t)\in R^n$ and $u(t)\in R^m$ are the state vector and control input vector, respectively, $A_0\in R^{n\times n}$ and $B\in R^{n\times m}$ are two constant matrices, $m\le n$, $g(x)$ is the nonlinear term, ΔA is the admissible parameter uncertainty , x_0 is the initial condition, and the following conditions are assumed.

Assumption 1: (uncertain condition) The admissible parameter uncertainty ΔA satisfies:

$$\Delta A=DF(t)E \ . \tag{7}$$

where D and E are two constant matrices and F(t) satisfies

$$F^T(t)F(t)\le I,\forall t \ . \tag{8}$$

Assumption 2: (nonlinear condition) The nonlinear term satisfies:

$$\lim_{x\to 0}\frac{\|g(x)\|}{\|x\|}=0 \text{ and } g(x)|_{x=0}=0 \ . \tag{9}$$

Infact, most of the chaotic system can be described as the form of system (6), such as Chen, Liu, Lü, Lorenz chaotic systems and some hyperchaotic systems composed of those chaotic system via linear or nonlinear feedback control.

Next, we present several important results for robust optimial stable control for system (6).

Definition ([27]). Consider the uncertain dynamic system (6). Suppose that there exist a control law

$$u(t)=-Kx(t)=-R^{-1}B^TPx(t) \ . \tag{10}$$

and a positive real number *J** such that for all admissible uncertainties, the closed system (6) is stable and the corresponding performance index (5) satisfies $J\le J^*=x(0)^TPx(0)$. Then, the system (6) is said to be robust optimal un+der control low (10). *J** is said to be a guaranteed cost, and *u*(t) is said to be a guaranteed cost control law.

Theorem 1 Consider the uncertain chaotic system (6) with the performance index (5). If there exist a positive definite matrix P=PT for all uncertainties holds

$$(A_0+\Delta A-BK)^T P+P(A_0+\Delta A-BK)+Q+K^T RK+M<0. \quad (11)$$

Where M is a positive definite sys-metric matrices .Then, the closed system is robust optimal with control law (10) and the optimal performance index (5) is $J\le J^*=x(0)^T Px(0)$.

Proof.Consider the Lyapunov function candidate $V=x^T Px$, then the time derivative of V along the trajectory of (6) is

$$\begin{aligned}\dot V &= \dot x^T Px+x^T P\dot x\\ &=[(A_0+\Delta A-BK)x+g(x)]^T Px+x^T P[(A_0+\Delta A-BK)x+g(x)]\\ &=x^T[(A_0+\Delta A-BK)^T P+P(A_0+\Delta A-BK)]x+2x^T Pg(x)\\ &<x^T[-Q-K^T RK]x-V_1 \qquad (12)\end{aligned}$$

Where $V_1=x^T Mx-2x^T Pg(x)$.

Note the nonlinear condition $\lim\limits_{x\to 0}\frac{\|g(x)\|}{\|x\|}=0$, that is, for $\forall\xi$, $\exists\delta>0$, if $\|x\|\le\delta$, $\frac{\|g(x)\|}{\|x\|}<\xi$. Namely, $\|g(x)\|<\xi\|x\|$.

$$2x^T Pg(x)\le 2\|x^T P\|\|g(x)\|=2\sqrt{x^T PPx}\,\|g(x)\|\le 2\sqrt{\lambda_{\max}(P^2)}\,\|x\|\|g(x)\|$$

$$x^T Mx\le\lambda_{\min}(M)\|x\|^2$$

So if $\|x\|\le\delta$, $V_1=x^T Mx-2x^T Pg(x)<-(\lambda_{\min}(M)-2\xi\sqrt{\lambda_{\min}(P^2)}\,\|x\|^2$.

Because $\lambda_{\min}(M)$ and $\lambda_{\min}(P^2)$ are both known number, so if $\xi<\frac{\lambda_{\min}(M)}{\lambda_{\min}(P^2)}$, $V_1<0$.

Thus,

$$\dot V<x^T[-Q-K^T RK]x<0. \quad (13)$$

Next, we verify the optimal cost index $J\le J^*=x(0)^T Px(0)$.

By integrating both sides of (15) from 0 to ∞ and noting that $V(x(t))\to 0$ when $t\to 0$, we have $J\le J^*=x(0)^T Px(0)$.

This completes the proof of Theorem 1.

Remark 1: Theorem 1 gives a sufficient condition for system (6). It should be noted that, the unknown parameters ΔA is not limited to the assumption of (7), but for all uncertainties for matrix A_0.

Remark 2: Theorem 2 gives a sufficient condition for system (6) and the limitation of unknown parameters ΔA in (7).

Theorem 2 Consider the uncertain nonlinear system (6) with the performance index (5). If there exist the positive definite matrix $X=X^T=P^{-1}$ and positive constant number ε such that the following matrix inequality holds

$$\begin{bmatrix} A_0X+XA_0+\varepsilon DD^T-BR^{-1}B^T & X\\ X^T & -(Q+M+\varepsilon^{-1}E^T E)\end{bmatrix}<0. \quad (14)$$

Then, the closed system is robust optimal under control law (10), and the optimal performance index is $J\le J^*=x(0)^T Px(0)$.

Proof. Substituting $X=X^T=P^{-1}$ into (14), then using Schur complement, we can get that

$$A_0^T P + PA_0 - PBR^{-1}B^T P + \varepsilon PDD^T P + \varepsilon^{-1}E^T E + Q + M < 0 . \quad (15)$$

Consider the Lyapunov function candidate $V = x^T Px$, then the time derivative of V along the trajectory of (6) is

$$\dot{V} = \dot{x}^T Px + x^T P\dot{x} = [(A_0 + \Delta A - BK)x + g(x)]^T Px + x^T P[(A_0 + \Delta A - BK)x + g(x)]$$

Note that

$$\begin{aligned}\Delta A^T P + P\Delta A &= (DF(t)E)^T P + PDF(t)E \\ &\le \varepsilon(PD)(PD)^T + \varepsilon^{-1}(F(t)E)^T(F(t)E) \\ &= \varepsilon PDD^T P + \varepsilon^{-1}E^T E\end{aligned}$$

So

$$\begin{aligned}\dot{V} &\le x^T[A_0^T P + PA_0 - 2PBR^{-1}B^T P + \varepsilon PDD^T P + \varepsilon^{-1}E^T E]x + 2x^T Pg(x) \\ &= x^T[-Q - K^T RK - M]x + 2x^T Pg(x) \\ &< x^T[-Q - K^T RK]x - V_1 \qquad (16)\end{aligned}$$

We can see that (16) is similar to (12), so the following process is omitted here.

Remark 3: Theorem 2 gives a sufficient condition for system (6) with the limitation of (5) in the form of LMI. It translates the problem of stability into the feasibility of the LMI, which can be solved by using the feasp command in the LMI toolbox within the MATLAB environment. Once a solution of the LMI is feasible, the robust optimal control law, that ensures the quadratic stability of the closed system (6), can be contracted readily.

4. Simulation results

In this section, the fourth-order Runge–Kutta method is used to solve PMSM chaotic system (4) with time step size 0.001 in all numerical simulations. The system parameters γ=25 and σ=5.46. Choose the initial conditions of the system $(x_1(0), x_2(0), x_3(0)) = (0.01, 0.01, 0.01)$.

First， we have to verify that the uncertain condition and nonlinear condition are both holded for system (4).

$$\lim_{x\to 0}\frac{\|g(x)\|}{\|x\|} = \lim_{e\to 0}\frac{\sqrt{x_2^2x_3^2 + x_1^2x_3^2}}{\sqrt{x_1^2 + x_2^2 + x_3^2}} \le \lim_{x\to 0}\frac{\sqrt{x_2^2x_3^2 + x_1^2x_3^2}}{\sqrt{x_1^2}} = \lim_{x\to 0}\sqrt{x_3^2 + x_3^2} = 0$$ and $g(x)|_{x=0} = 0$.

$$\Delta A = DF(t)E \quad , \quad F(t) = diag(rand(1), rand(1), rand(1)) \quad , \quad E = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \gamma \\ 0 & \sigma & -\sigma \end{bmatrix} ,$$

$D = diag(0, 0.3, 0.3)$ .Thus, we can design the controller according to Theory 3.

The control parameters $\varepsilon = 0.1$, $R = I^{2\times 2}$, $Q = M = I^{3\times 3}$. The control method takes effect after t=25 s.

1) Without considering uncertain parameters, that is $\Delta A = 0$, according to Theorem 2, we can get:

$$X = \begin{bmatrix} 0.0006 & 0 & 0 \\ 0 & 0.0747 & 0.2174 \\ 0 & 0.2174 & 1.0175 \end{bmatrix}, \; K = \begin{bmatrix} 0.6488 & 0 & 0 \\ 0 & 74.7336 & 217.3513 \end{bmatrix}$$

and the optimal cost index is $J^* = 0.1528$.

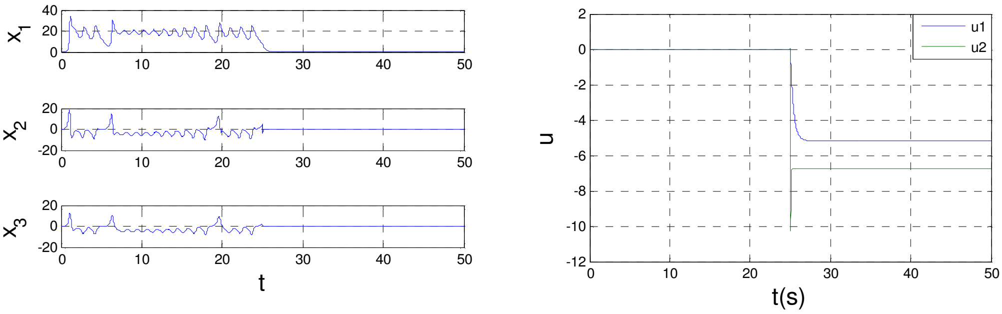
The state trajectories and control inputs for the controlled PMSM chaotic system disregarding the unknown parameters are shown in Fig. 2.

2) The uncertain parameters are assumed as the same as (7), according to Theorem 3, we can see the following result which satisfied LMI (13):

$$X=\begin{bmatrix}0.8614 & 0 & 0\\ 0 & 0.0350 & -0.0071\\ 0 & -0.0071 & 0.0024\end{bmatrix},\ K=\begin{bmatrix}1.610 & 0 & 0\\ 0 & 68.3730 & 197.4428\end{bmatrix}$$

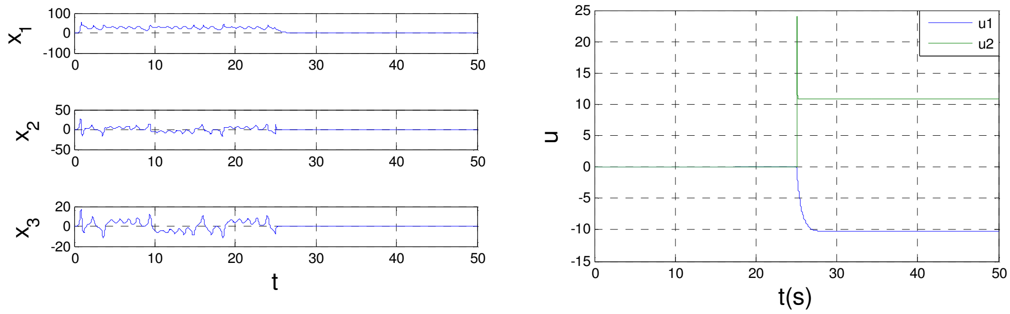
and the optimal cost index is $J^*=0.1444$.

Fig. 3 demonstrates the simulation results of the proposed controller for PMSM chaotic system with known parameters.



(a) (b)

Fig. 2 The performance of the system with the proposed controller (without considering uncertain parameters) (a) State trajectories (b) Control inputs



(a) (b)

Fig. 3 The performance of the system with the proposed controller (considering uncertain parameters) (a) State trajectories (b) Control inputs

We can see from Fig. 2 and Fig. 3 that the closed system (considering the uncertain parameters or not) is both stabilized to their diresed equilibrium point quickly, and the control inputs are continuous and smooth. From the simulation results, is shows that the obtained theoretic results are feasible and efficient for controlling PMSM chaotic system with unknown parameters.

5. Conclusion

In this paper, a novel robust optimal control method is developed for a PMSM chaotic system. Compared with other controllers, the one presented in this paper have some advantages such as simple structure, strong robustness and the control gains can be obtained easily based on LMI technique. The effectiveness of this proposed control method has been validated by numerical simulation in Section 4. Moreover, this method is a unified control scheme for a class of chaotic and hyper-chaotic systems, such as Lorenz, Chen, Lü, Liu chaotic systems and some hyper-chaotic systems constructed from them.

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References

[1] Chau, K.T. and Z. Wang, Chaos in eletric drive system: analysis, control and application. 2011, Singapore: John Wiley & Sons Inc.
[2] Henmati, N., Strange attractors in brushless DC motors. IEEE Trans. Circuit. Syst- I, 1994. 49(1): 40-45.
[3] Li, Z., et al., Bifurcations and chaos in a permanent-magnet synchronous motor. IEEE Trans. Circuit. Syst- I, 2002. 49(3): 383-387.
[4] Ren, H.P., D. Liu, and J. Li, Delay feedback control of chaos in permanent magnet synchronous motor. Proceedings of the CSEE, 2003. 23(6): 175-178.
[5] Wu, Z.Q. and F.X. Tan, Passivity control of permanent magnet synchronous motor chaotic systems. Proceedings of the CSEE, 2006. 26(18): 159-163.
[6] Zhang, X.H., et al., Inverse system control of chaos in permanent magnet synchronous motors. Electric machines and control, 2010. 14(10): 73-77.
[7] Ren, H.P. and D. Liu, Nonlinear feedback control of chaos in permanent magnet synchronous motor. IEEE Trans. Circuit. Syst- I, 2006. 53(1): 45-50.
[8] Zribi, M., A. Oteafy, and N. Smaoui, Controlling chaos inthe permanent magnet synchronous motor. Chaos Solitons Fractals, 2009. 41: 1266-1276.
[9] Ataei, M., A. Kiyoumarsi, and B. Ghorbani, Control of chaos in permanent magnet synchronous motor by using optimal Lyapunov exponents placement. Phys. Lett. A, 2012. 374: 4226-4230.
[10] Wei, D.Q., et al., Controlling chaos in permanent magnet synchronous motor based on the differential geometry method. Acta Phys. Sin., 2006. 55(1): 54-59.
[11] Wei, D.Q., et al., Adaptive controlling chaos permanent magnet synchronous motor based on the LaSalle theory. Acta Phys. Sin., 2009. 58(3): 6026-6029.
[12] Choi, H.H., Adaptive control of a chaotic permanent magnet synchronous motor. Nonlinear Dyn., 2012. 69: 1311-1322.
[13] Wei, D.Q. and X.S. Luo, Passive adaptive control of chaos in synchronous reluctance motor. Chin. Phys. B, 2008. 17(1): 92-97.
[14] Yu, J.P., B. Chen, and H.S. Yu, Fuzzy approximation based adaptive control of the chaotic permanent magnet synchronous motor. Nonlinear Dyn., 2012. 69: 1479-1488.
[15] Huang, C.F., et al., The design of quasi-sliding mode control for a permanent magnet synchronous motor with unmatched uncertainties. Comput. Math. Appl., 2012. 64(5): 1036-1043.
[16] Wei, D.Q. and B. zhang, Controlling chaos in permanent magnet synchronous motors based on finite-time stability theory. Chin. Phys. B, 2009. 18(4): 1399-1403.
[17] Li, D., et al., Impulsive control for permanent magnet synchronous motor with parameters uncertainties. Chin. Phys. Soc., 2008. 17(5): 1678-1684.
[18] Li, D., et al., Fuzzy control of chaos in permanent magnet synchronous motor with parameter uncertainties. Acta Phys. Sin., 2009. 58(3): 1432-1440.
[19] Wang, Z.Y., Y.L. Cai, and W.J. Liu, Optimal control of chaotic motions in the liquid regulation system. Control Theory and Application, 1999. 16(2): 258-261.
[20] Lenci, S. and G. Rega, Optimal control of homoclinic bifurcation: theoretical treatment and practical reduction of safe basin erosion in the Helmholtz oscillator. J. Vib. Control, 2003. 9(3-4): 281-315.
[21] Marat, R. and J. Mamoel Balthazar, On an optimal control fordesign for Rossler system. Phys. Lett. A, 2004. 333(3): 241-245.
[22] Rafikov, M. and J.M. Balthazar, On control and synchronization in chaotic and hyperchaotic systems via linear feedback control. Commun. Nonlinear Sci. Numer. Simulat., 2008. 13: 1246-1255.
[23] Grzybowski, J.M.V., M. Rafikov, and J.M. Balthazar, Synchronization of the unified chaotic system and application in secure communication. Commun. Nonlinear Sci. Numer. Simulat., 2009. 14: 2793-2806.
[24] Chavarette, F.R., et al., A reducing of a chaotic movement to a periodic orbit, of a micro-electro-mechanical system, by using an optimal linear control design. Commun. Nonlinear Sci. Numer. Simulat., 2009. 14: 1844-1853.
[25] Zhu, S.P., F.C. Qian, and D. Liu, Optimal control for uncertainty dynamic chaotic systems. Acta Phys. Sin., 2010. 59(4): 2250-2255.
[26] Abedini, M., R. Vatankhah, and N. Assadian, Stabilizing chaotic system on periodic orbits using multi-interval and modern optimal control strategies. Commun. Nonlinear Sci. Numer. Simulat., 2012. 17: 3832-3842.
[27] Xue, A.K., Robust optimal control theory and its application. 2004, Beijing: Science Press.