

Regular paper

## High Dynamic Torque Capability With Efficiency Optimization Control of Induction Motor Drives

*The objective of this paper is to present an analysis of how to get high performance of an induction motor drive for both dynamic and steady state operation. In order to improve a good dynamic response in transient (minimize the speed drop), an algorithm for a maximum torque capability of the available maximum inverter current and voltage is presented. When the rotor speed reaches its reference, a loss minimization algorithm is developed to improve high efficiency. Simulation studies show the performance of the proposed work.*

**Keywords:** Induction motor, maximum torque control, efficiency optimization, inverter constraints.

### 1. INTRODUCTION

In addition to its advantages, such as speed capability, mechanical robustness, cheapness and ease of maintenance, when used with a field oriented control scheme, the induction machine is the most widely used electrical machines. Over the two last decades, extensive work has been done to obtain high performance for both dynamic and steady state operation. The shortest duration of the transient ( minimum speed drop ) is the objective of a variable speed-control system for high productivity, and results if an algorithm for the maximum-torque during transient operation is applied under voltage and stator current constraints. Therefore, the maximum torque generation problem reduces to the determination of the optimal subdivision of the available maximum current into the flux-producing and torque-producing stator current components.

Several studies have been reported that minimises time response. In [3], the authors suggest to retain the existing value of the flux producing current to its rated value and use all of the available inverter current capability to increase the torque producing current. The drawback of this method is the slow transient response. Another possibility in [1] propose to apply at first all of the available current to the flux-producing current component, so that dynamic torque is initially zero. Once when the forced flux build-up is completed, all of of the

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available inverter current is switched into the torque-producing current component. This method requires a delay-time between the two phases. Hence the minimum time speed control is not guaranteed. An alternative method for the maximum torque generation in transient operation under unsaturated conditions is described in [2]. This method is simple to implement and is widely used in the existing literature. Note that None of the techniques described previously attempts to account for the main inductance saturation in the machine. On the other hand, in the steady state, the key to solving the problem of maximizing the IM efficiency is to obtain the better balance between different types of motor losses. The previous studies on efficiency maximization techniques may be divided into two categories [4-5]:

- Loss model controller (LMC).
- Search controller (SC).

Both the two methods minimize the motor losses but in different ways. The LMC method calculates the optimum of the objective function (which is an analytical expression representing the total losses of the machine). The fast determination of the optimum variables is the merit of this method, but it is sensitive to parameter variations, hence if the approach is not based on an on-line estimation of the parameters then it is likely that the method may offer only sub-optimal solution if the parameters of the machine change ( due to saturation, temperature variations, skin effect, etc..). The SC technique depends on the exact measurement of the input power and converge slowly to the true optimum variables. The advantage of this approach is no-dependance on parameter variations. However, some disadvantages appear in practice, such as continuous disturbance in the torque, slow convergence because it has no idea about the optimal magnitude of variables at the beginning of search process, difficulties in tuning the algorithm for a given application. For these reasons, this is not a good method in industrial drives .

In this paper, we investigate how to track the reference speed as fast as possible in the transient stage using a maximum torque capability under saturated conditions, and using a minimum loss control strategy in the steady state to improve high efficiency of the drive system including induction motor losses and inverter losses.

## **2. LOSS MODELING OF THE DRIVE SYSTEM**

### **2.1 Induction machine loss**

- **Core losses**

The stator core losses due to fundamental frequency mutual flux  $\phi$  (air-gap flux) consists of eddy-current loss and hysteresis loss, and they are given by Steinmetz's formula as:

$$P_{cs} = K_e f_s^2 \phi^2 + K_h f_s \phi^2$$

where  $K_e$  and  $K_h$  are eddy-current and hysteresis coefficients and  $f_s$  is the stator flux frequency.

The stator equivalent core loss resistance is determined from the classical experimental no-load test data by [6]:

$$R_{fs} = a_1 f_s + a_2 f_s^2 \tag{1}$$

- **Copper losses**

Stator copper losses :

$$P_{js} = R_s (i_{ds}^2 + i_{qs}^2) \tag{2}$$

Rotor copper losses:

$$P_{jr} = R_r (i_{dr}^2 + i_{qr}^2) = \frac{R_r}{(1 + \sigma_r)^2} \left( \left( \frac{\phi_{dr}}{L_m} - i_{ds} \right)^2 + i_{qs}^2 \right) \tag{3}$$

- **Stray load losses**

While the copper losses, core losses and friction and windage losses could be obtained from the classical tests, such as no-load and blocked rotor test, The stray load losses are much harder to determine.

The stray losses are mainly attributed to the non-uniform current distribution (in both stator and rotor) and the distortion of the magnetic flux by the load current, respectively [6-7]. Since the rotor current in a squirrel cage induction motor is not measurable, it is common practice to express the stray losses as a function of the stator current [6].

The stator stray-loss  $P_{st}$  is taken into account by the inclusion of an equivalent stator stray-loss resistance  $R_{st}$  in series with the stator phase resistance:

$$P_{st} = K_{st} (K_h f_s + K_e f_s^2) I_s^2 = R_{st} I_s^2 \tag{4}$$

$$R_{st} = K_{st} (K_h f_s + K_e f_s^2) = a_3 f_s + a_4 f_s^2$$

Where  $K_{st}$  is the stray loss constant

- **Friction and windage loss**

The main reasons of mechanical losses appearing are friction and windage losses. One can separate some components of these losses as follows:

1. Friction losses in bearings
2. Windage losses of outside fan
3. Friction air losses of rotor and windage losses of two internal fans (casted with rotor rings).

The friction and windage losses are modeled as a function of motor speed. It can be expressed as:

$$P_m = a_5 \Omega^2 \quad (5)$$

## 2.2 Inverter loss model

The inverter loss calculation is based on measurement of the diode and transistor conduction voltages and on measurements of the following switching energies:

Diode turn-off, transistor turn-on and transistor turn-off. The approximate inverter loss as function of stator current is given by [7]:

$$W_{inv} = a_6 (i_{ds}^2 + i_{qs}^2) + a_7 \sqrt{i_{ds}^2 + i_{qs}^2} \quad (6)$$

$a_6$  and  $a_7$  coefficients determined by the electrical characteristics of a switching element. In this work we have ( $a_6 = 0.0606$ ,  $a_7 = 5.49$ ).

## 2.3 Induction motor model

The induction motor model used under field oriented control FOC can be expressed in the synchronously rotating d-q reference frame as follows:

$$\begin{aligned} V_{ds} = & \left( R_s + R_{st} + \frac{\sigma_r R_{fs}}{1 + \sigma_r} \right) i_{ds} + \sigma L_s \frac{di_{ds}}{dt} - \sigma L_s \omega_s i_{qs} \\ & + (1 - \sigma)(1 + \sigma_s) \frac{d\phi_{dr}}{dt} + \frac{R_{fs}}{(1 + \sigma_r)L_m} \phi_{dr} \end{aligned} \quad (7)$$

$$\begin{aligned} V_{qs} = & \left( R_s + R_{st} + \frac{\sigma_r R_{fs}}{1 + \sigma_r} \right) i_{qs} + \sigma L_s \frac{di_{qs}}{dt} + \sigma L_s \omega_s i_{ds} \\ & + (1 - \sigma) \cdot (1 + \sigma_s) \cdot \omega_s \cdot \phi_{dr} \end{aligned}$$

$$T_{em} = p(1-\sigma)(1+\sigma_s)\phi_{dr}\dot{i}_{qs} \quad (8)$$

$$T_r \frac{d\phi_{dr}}{dt} + \phi_{dr} = L_m \dot{i}_{ds} \quad (9)$$

### 3. MAXIMUM TORQUE CAPABILITY

Delivering a maximum torque by induction machine in the transient state is very important for high dynamic performance. The developed torque is constrained by allowable voltage and current ratings of the machine, as well as the inverter.

In the steady state, The electromagnetic torque can be given as:

$$T_{em} = p(1-\sigma)(1+\sigma_s)L_m\dot{i}_{ds}\dot{i}_{qs} \quad (10)$$

- **Current limit**

The maximum stator current  $I_{\max}$  is usually limited to 1.5 times the rated current to provide higher acceleration torque during transients.

$$0 \leq \sqrt{\dot{i}_{ds}^2 + \dot{i}_{qs}^2} \leq I_{\max} \quad (11)$$

- **Voltage limit**

The maximum stator voltage  $V_{\max}$  is determined from the available dc-link voltage  $V_{dc}$  and pulse width modulation strategy, then the stator voltage magnitude is constrained by:

$$0 \leq \sqrt{V_{ds}^2 + V_{qs}^2} \leq V_{\max} \quad (12)$$

with  $V_{\max} = \frac{2}{\pi}V_{dc}$ , and  $\frac{2}{\pi}$  is the maximum achievable modulation index of a three-phase inverter .[3].

- **Saturation effect**

Rotor flux variation given with (9) , is valid only under the assumption of linear magnetic circuit. Such an assumption in real and exact cases is not admissible because inductances are non-linear functions of currents. It is therefore necessary to modify the equation of the rotor flux in (9), so that it accounts for the main flux saturation.

An acceptable approximation is that, two inductances are defined: an unsaturated inductance  $L_{s0}$  which applies for stator current lower than  $I_{s0}$ , and a saturated one for higher  $I_s$  current.

$$L_s = \begin{cases} 0.6 \text{ H} & I_s < 0.8A \\ \frac{1.33}{1.44 + |I_s|} \text{ H} & I_s > 0.8A \end{cases} \quad (13)$$

#### 4. OPTIMAL CURRENTS FOR MAXIMUM TORQUE CAPABILITY IN TRANSIENT STATE

The basic idea of the next section is to determine the reference currents  $i_{ds}^*$  and  $i_{qs}^*$  providing maximum torque under constraints (11) and (12). The steady state voltage equations for the rotor flux oriented induction machine in the rotating  $d$ - $q$  reference frame can be derived as:

$$\begin{aligned} V_{ds} &= (R_s + R_{st} + R_{fs}) i_{ds} - \sigma L_s \omega_s i_{qs} \\ V_{qs} &= \left( R_s + R_{st} + \frac{\sigma_r R_{fs}}{1 + \sigma_r} \right) i_{qs} + L_s \omega_s i_{ds} \end{aligned} \quad (14)$$

Substitution of (14) into (12) will result in another condition current limit as:

$$\begin{aligned} &\left[ (R_s + R_{st} + R_{fs})^2 + L_s^2 \omega_s^2 \right] i_{ds}^2 \\ &+ \left[ \left( R_s + R_{st} + \frac{\sigma_r}{1 + \sigma_r} R_{fs} \right)^2 + \sigma^2 L_s^2 \omega_s^2 \right] i_{qs}^2 \\ &+ \left[ 2L_s \omega_s \left( (R_s + R_{st})(1 - \sigma) + \left( \frac{\sigma_r}{1 + \sigma_r} - \sigma \right) R_{fs} \right) \right] i_{ds} i_{qs} \leq V_{\max}^2 \end{aligned} \quad (15)$$

if we denote

$$\begin{aligned} A &= (R_s + R_{st} + R_{fs})^2 + L_s^2 \omega_s^2 \\ B &= 2L_s \omega_s \left( (R_s + R_{st})(1 - \sigma) + \left( \frac{\sigma_r}{1 + \sigma_r} - \sigma \right) R_{fs} \right) \\ C &= \left( R_s + R_{st} + \frac{\sigma_r}{1 + \sigma_r} R_{fs} \right)^2 + \sigma^2 L_s^2 \omega_s^2 \end{aligned}$$

The solution of (11) and (15) can be summarized as follows: ( for more details see [2])

When  $I_{\max} \leq \frac{\sqrt{2} V_{\max}}{\sqrt{A+B+C}}$  we get:

$$i_{ds}^* = i_{qs}^* = \frac{I_{\max}}{\sqrt{2}} \quad (16)$$

When  $\frac{\sqrt{2} V_{\max}}{\sqrt{A+B+C}} \leq I_{\max} \leq \frac{V_{\max}}{\sqrt{A' \sin^2 \alpha + C' \cos^2 \alpha}}$  we get:

$$\begin{aligned} i_{ds}^* &= \frac{\sqrt{V_{\max}^2 - C' I_{\max}^2}}{\sqrt{A' - C'}} \cos \alpha - \frac{\sqrt{A' I_{\max}^2 - V_{\max}^2}}{\sqrt{A' - C'}} \sin \alpha \\ i_{qs}^* &= \frac{\sqrt{V_{\max}^2 - C' I_{\max}^2}}{\sqrt{A' - C'}} \sin \alpha + \frac{\sqrt{A' I_{\max}^2 - V_{\max}^2}}{\sqrt{A' - C'}} \cos \alpha \end{aligned} \quad (17)$$

where

$$\begin{aligned} \alpha &= \frac{1}{2} \arctg \left[ \frac{B}{A-C} \right] \\ A' &= \frac{A+C + \sqrt{(A-C)^2 + B^2}}{2} \\ C' &= \frac{A+C - \sqrt{(A-C)^2 + B^2}}{2} \end{aligned}$$

When  $I_{\max} \geq \frac{V_{\max}}{\sqrt{A' \sin^2 \alpha + C' \cos^2 \alpha}}$  we get:

$$\begin{aligned} i_{ds}^* &= V_{\max} \left[ \frac{\sqrt{C}}{\sqrt{A(B + \sqrt{4AC})}} \right]^{1/2} \\ i_{qs}^* &= V_{\max} \left[ \frac{\sqrt{A}}{\sqrt{C(B + \sqrt{4AC})}} \right]^{1/2} \end{aligned} \quad (18)$$

According to the derived solutions of stator current components of (16), (17) and (18), the maximum torque  $T_{\max}$  can be delivered using:

$$T_{\max} = p(1-\sigma)(1+\sigma_s) L_m i_{ds}^* i_{qs}^* \quad (19)$$

To demonstrate the merit of the maximum torque generation in transient, the simulation result of both developed torques without load is shown in Figure 1. providing a minimum time for transient state.

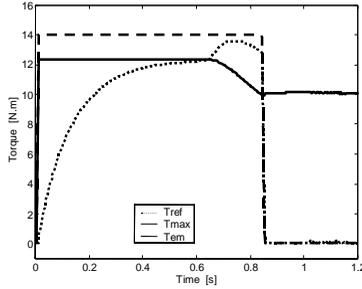


Figure 1. Minimum time control.

## 5. LOSS MINIMIZATION STRATEGY OF THE DRIVE SYSTEM

Several minimum-loss control schemes which uses rotor flux as primary control variable have been reported previously. The control law described in these studies is not practical because the flux is difficult to obtain in real time [5-7]. In the next section, a minimum-loss control law that uses the direct component of stator current as the principle control variable is described.

By minimizing the total loss  $W_{tot}$ , which is the sum of motor losses  $W_{mot}$  and the inverter losses  $W_{inv}$ , it is possible to maximize the efficiency of the total drive system.

The inverter ac-side and dc-side instantaneous powers can be written as:

$$P_{in} = V_{dc} I_{dc} = W_{inv} + V_{ds} i_{ds} + V_{qs} i_{qs} \quad (20)$$

Where  $V_{dc}$  and  $I_{dc}$  are the instantaneous dc-link voltage and current respectively.

The total power loss  $W_{tot}$  of the drive system can be given by:

$$\begin{aligned} W_{tot} &= W_{mot} + W_{inv} \\ W_{mot} &= P_{js} + P_{jr} + P_{fs} + P_{st} + P_m = P_{in} - P_{out} - W_{inv} \end{aligned} \quad (21)$$

Therefore, the total loss of the drive system can be represent as:

$$W_{tot} = (R_s + R_{st})(i_{ds}^2 + i_{qs}^2) + \frac{R_{fs}}{1 + \sigma_r} [\sigma_r i_{qs}^2 + (1 + \sigma_r) i_{ds}^2] \quad (22)$$

$$+ \frac{R_r}{(1 + \sigma_r)^2} i_{qs}^2 + a_5 \Omega_r^2 + a_6 (i_{ds}^2 + i_{qs}^2) + a_7 \sqrt{i_{ds}^2 + i_{qs}^2}$$

This means that  $W_{tot}$  is a function of d-axis component of stator current. Figure 2 shows this relationship for various load torques.

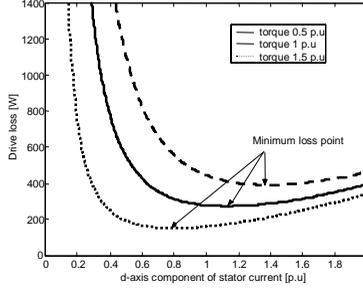


Figure 2. Total loss of the drive system versus  $i_{ds}$ .

The loss minimization condition of the drive system at steady state ( $T_{em}$  and  $\Omega$  constant) gives:

$$\frac{\partial W_{tot}}{\partial i_{ds}} = 0 \quad (23)$$

which yields:

$$i_{ds}^* = \frac{I_{max}}{\sqrt{2}} \sqrt{\frac{F + 2EI_{max}}{F + (D + E)I_{max}}} \quad (24)$$

In (24),  $D$ ,  $E$  and  $F$  are given by:

$$D = R_s + R_{st} + R_{fs} + a_6$$

$$E = R_s + R_{st} + \frac{R_r}{(1 + \sigma_r)^2} + \frac{\sigma_r R_{fs}}{(1 + \sigma_r)} + a_6 \quad (25)$$

$$F = a_7$$

The relationship that maximizes the efficiency as function of motor speed and motor parameters can be obtained as:

$$i_{ds}^* = K_{opt} i_{qs}^* \quad (26)$$

where the optimal current ratio  $K_{opt}$  is given by:

$$K_{opt} = \sqrt{\frac{F + 2EI_{\max}}{F + 2DI_{\max}}} \tag{27}$$

Thus, for minimum loss, the optimal current ratio is constant at a given speed and depends only on the motor parameters.

The electromagnetic torque can be derived as:

$$T_{em} = p(1 - \sigma)L_s K_{opt} (i_{qs}^*)^2 \tag{28}$$

The appropriate d-axis component of stator current given in (24) for various load torques is plotted in Figure 3. It is affected by motor speed because core loss, mechanical loss and stray load loss are taken into account.

Figure 4. shows a map of total drive loss versus torque for various driving conditions.

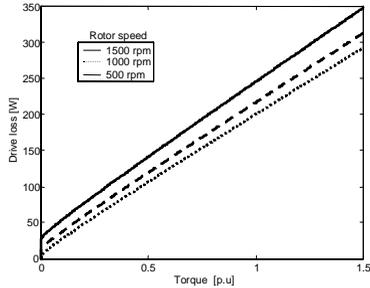
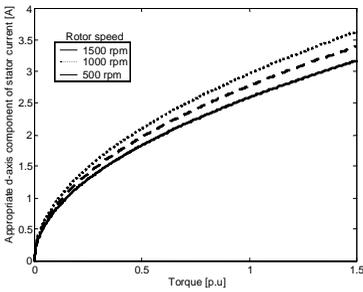


Figure 4. Map of the drive loss versus torque.

Figure 3. d-axis component of stator current versus torque.

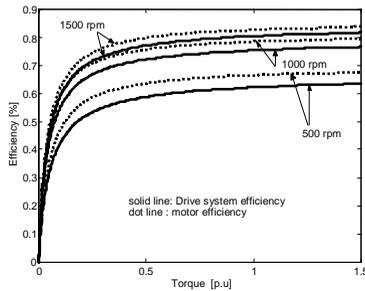


Figure 5. Map of the efficiency for both drive system and motor versus torque.

Figure 5. shows an efficiency map versus torque. It is clear that the proposed method is superior to the conventional method which keeps rotor flux constant at its rated value over a wide range of torques.

## 6.- SYSTEM CONFIGURATION

The system configuration of the proposed study can be divided into two parts:

1. The first part deals with how to achieve a minimum-time speed response in the transient state by comparing the reference torque computed from the mechanical equation of the induction motor with the maximum torque given by (19).

When reference torque  $T_{ref}$  is greater than the maximum torque  $T_{max}$  (Figure 1), reference currents can be obtained from (16), (17) and (18).

2. In the second part, after the rotor speed reaches its reference value, a minimum loss speed control is applied to obtain high efficiency of the drive system. The reference currents should be determined from (24) and (26).

## 7. SIMULATION RESULTS

The proposed algorithm is tested by simulation. The motor initially operates under no-load conditions, at  $t = 3$  s the load torque is stepped to 3N.m.

As can be seen from Figure 6. and Figure 7. the proposed study deals with the two stages. In transient state, the method leads to by far the smallest drop in speed, consequently, the time interval needed for the motor to return to the reference value is the shortest. When the speed reaches its reference value (150rd/s), a minimum loss speed control is applied to improve high efficiency of the drive system.

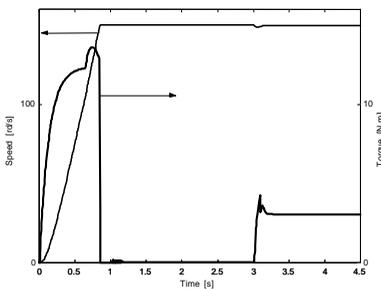


Figure 6. Response of speed and torque of the drive to load impact condition.

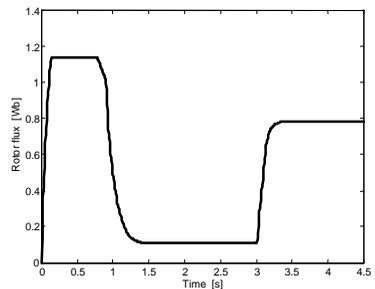


Figure 7. Rotor flux versus time

## 8. CONCLUSION

A minimum-time minimum-loss control strategy for induction motor drive is presented which has the straightforward goal of minimizing the speed drop in transient state, i.e. track the reference speed as fast as possible over the entire speed under allowable constraints on current and voltage, and provide maximum efficiency of the drive system when the speed return to its reference.

Detailed derivation of the optimal current sharing algorithm is illustrated for the two stages and some simulation results have been carried out providing a superior behaviour resulting in a smaller speed drop and a high efficiency of the system drive.

## APPENDIX

$$R_s = 8\Omega, R_r = 3.1\Omega, L_s = L_r = 0.47\text{H}, L_m = 0.443\text{H},$$
$$p = 2, J = 0.06\text{ kg.m}^2, f = 0.0042\text{ N.m.s},$$
$$P_n = 1.1\text{KW}, T_n = 7\text{N.m}, N_n = 1430\text{tr/min}, I_n = 3.4\text{ A}.$$

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